Christ Church Grammar School

MATHEMATICS METHODS Year 11
Section One:
Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                  correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to
ensure that you do not have any unauthorised notes or other items of a non-personal
nature in the examination room. If you have any unauthorised material with you, hand it
to the supervisor **before** reading any further.

Instructions to candidates

1.   Write your answers in this Question/Answer Booklet.

2.   Answer all questions.

3.   You must be careful to confine your response to the specific question asked and to follow any instructions that are specific to a particular question.

4.   Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5.   **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6.   It is recommended that **you do not use pencil**, except in diagrams.

Question 1 (4 marks)

A curve $y = f(x)$ passes through the point (3,2) and has $f'(x) = 3 - x^2$.

Calculate the following:

(a)   The equation of the curve. (3 marks)

$$f'(x) = 3 - x^2$$

$$f(x) = 3x - \frac{1}{3}x^3 + C \qquad \checkmark \text{(for equation of curve in terms of } C)$$

use (3, 2)

$$2 = 3 \times 3 - \frac{1}{3}(3)^3 + C$$
$$2 = 9 - 9 + C$$
$$C = 2 \qquad \checkmark \text{(for correct value of constant)}$$

So equation is

$$y = 3x - \frac{1}{3}x^3 + 2 \qquad \checkmark \text{(for final equation)}$$

Note: can also have
$$f(x) = 3x - \frac{1}{3}x^3 + 2$$
as answer

(b)   The value of y when $x = -1$. (1 mark)

at $x = -1$. $y = 3 \times (-1) - \frac{1}{3}(-1)^3 + 2$

$$= -3 + \frac{1}{3} + 2$$

$$= -\frac{2}{3} \qquad \checkmark \text{(for final answer)}$$

Note: can also have $f(x) = -\frac{2}{3}$ as answer

See next page

Question 2   **(6 marks)**

S_n, the sum of the first n terms of an arithmetic sequence, is given by:

$$S_n = 17n - 3n^2$$

(a) Determine the sum of the first 10 terms of the arithmetic sequence.   (1 mark)

$$S_{10} = 17 \times 10 - 3 \times 10^2$$
$$= 170 - 300 = -130 \qquad \checkmark (\text{for final answer})$$

(b) Show that the first and second terms of the arithmetic sequence are 14 and 8
respectively.   (3 marks)

$$S_1 = 17 \times 1 - 3 \times 1^2 = 14 = \text{1st term}$$

$\checkmark$ (gives first term in sequence)

$$S_2 = 17 \times 2 - 3 \times (2)^2 = 22$$

$\checkmark$ (for sum of first two terms of sequence)

$$\therefore \text{2nd term} = 22 - 14 = 8$$

$\checkmark$ (gives second term in sequence)

(c) Give a simplified expression, in terms of n, for the nth term of the arithmetic
sequence.   (2 marks)

Sequence is an A.P $14, 8, 2, -\cdots$

common difference $= -6$

$\checkmark$ (for correct common difference)

$$n\text{th term} = a + (n-1)d$$
$$= 14 + (n-1) \times (-6)$$
$$= 14 - 6n + 6$$
$$= 20 - 6n$$

$\checkmark$ (for final simplified answer)

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Question 3 (5 marks)

Two particles A and B are moving along a straight path so that their displacements x_A and x_B *metres* relative to the origin O at time t *seconds* ($t \geq 0$) are given by $x_A = 3t^2 + 5t - 10$ and $x_B = -2t^2 + 15t + 5$ respectively. Calculate the speeds of the two particles at the instant they collide.

for collision:

$$3t^2 + 5t - 10 = -2t^2 + 15t + 5$$

✓ (for giving correct equation for collision)

$$5t^2 - 10t - 15 = 0$$
$$t^2 - 2t - 3 = 0$$
$$(t-3)(t+1) = 0$$
$$t = 3, -1 \quad \left(\text{discount } t = -1 \text{ as } t \geq 0\right)$$

So collision at $t = 3$

✓ (for $t = 3$ and discounting $t = -1$, at collision)

$$V_A = \frac{dx_A}{dt} = 6t + 5$$ ✓ (for $\frac{dx_A}{dt}$ expression)

$$V_B = \frac{dx_B}{dt} = -4t + 15$$ ✓ (for $\frac{dx_B}{dt}$ expression)

at $t = 3$

$$\frac{dx_A}{dt} = 6 \times 3 + 5 = 23$$

$$\frac{dx_B}{dt} = -4 \times 3 + 15 = 3$$

∴ speed of A is 23 m/s
speed of B is 3 m/s

✓ (for both speeds)

End of questions

Additional working space

Question number:_____